Mathematical Derivation in Statistical Computing

What is it?

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R packages

- R is a software mainly used by statisticians
- Object-oriented programming environment
- Basic software vs. Packages

What is it?

- CARramps is an R package that uses the **graphical processing unit (GPU)** of a computer to draw random samples for certain types of statistical models.

GPU’s

- Used to display desktop image
- Parallel computing

What is it?

- CARRramps is an R package that uses the graphical processing unit (GPU) of a computer to draw random samples for **certain types of statistical models**.

Models

- Best used for lattice data
- Example: satellite images
- Accounts for measurement error

Is there any math involved?

You bet your bottom dollar there is!!

$$p(\tau_{tot}^2, \boldsymbol{s}|\boldsymbol{y}) \propto \left(\prod_{j=0}^{F-1} s_j^{(\alpha_j - 1)}\right)\left(\prod_{i=1}^{n-k} \gamma_i\right)^{\frac{1}{2}} \left((\tau_{tot}^2)^{\sum_{j=0}^{F-1} \alpha_j + \frac{(n-k)}{2} - 1}\right)$$

$$* \, exp\left[-\tau_{tot}^2 \left(\sum_{j=0}^{F-1} (s_j \beta_j) + \sum_{i=1}^{n-k} (By)_i^2 \gamma_i\right)\right]$$

Want to find: $p(\boldsymbol{s}|\boldsymbol{y})$

$$p(\tau_{tot}^2, \boldsymbol{s}|\boldsymbol{y}) = p(\tau_{tot}^2 \,|\, \boldsymbol{s}, \boldsymbol{y}) * p(\boldsymbol{s}|\boldsymbol{y})$$

$$p(\boldsymbol{s}|\boldsymbol{y}) = \frac{p(\tau_{tot}^2, \boldsymbol{s}|\boldsymbol{y})}{p(\tau_{tot}^2 \,|\, \boldsymbol{s}, \boldsymbol{y})}$$

Sidebar

$$Gamma(x; r, \lambda) = \frac{1}{\Gamma(r)} \lambda^r x^{r-1} e^{-\lambda x}$$

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$$p(\tau_{tot}^2, \mathbf{s} | \mathbf{y}) \propto \left(\prod_{j=0}^{F-1} s_j^{(\alpha_j - 1)} \right) \left(\prod_{i=1}^{n-k} \gamma_i \right)^{\frac{1}{2}} \left((\tau_{tot}^2)^{\sum_{j=0}^{F-1} \alpha_j + \frac{(n-k)}{2} - 1} \right)$$

$$* \, exp \left[-\tau_{tot}^2 \left(\sum_{j=0}^{F-1} (s_j \beta_j) + \sum_{i=1}^{n-k} (By)_i^2 \gamma_i \right) \right]$$

$$p(\tau_{tot}^2 | \, \mathbf{s}, \mathbf{y}) = Gamma \left(\sum_{j=0}^{F-1} \alpha_j + \frac{(n-k)}{2}, \sum_{j=0}^{F-1} (s_j \beta_j) + \sum_{i=1}^{n-k} (By)_i^2 \gamma_i \right)$$

$$p(\boldsymbol{s}|\boldsymbol{y}) = \frac{p(\tau_{tot}^2, \boldsymbol{s}|\boldsymbol{y})}{p(\tau_{tot}^2 |\, \boldsymbol{s}, \boldsymbol{y})}$$

$$p(\tau_{tot}^2 |\, \boldsymbol{s}, \boldsymbol{y}) \sim Gamma(x; r, \lambda) = \frac{1}{\Gamma(r)} \lambda^r x^{r-1} e^{-\lambda x}$$

$$p(\tau_{tot}^2, \boldsymbol{s}|\boldsymbol{y}) \propto \left(\prod_{j=0}^{F-1} s_j^{(\alpha_j - 1)}\right)\left(\prod_{i=1}^{n-k} \gamma_i\right)^{\frac{1}{2}} \left((\tau_{tot}^2)^{\sum_{j=0}^{F-1} \alpha_j + \frac{(n-k)}{2} - 1}\right)$$

$$* \exp\left[-\tau_{tot}^2 \left(\sum_{j=0}^{F-1} (s_j \beta_j) + \sum_{i=1}^{n-k} (By)_i^2 \gamma_i\right)\right]$$

$$p(\boldsymbol{s}|\boldsymbol{y}) \propto \frac{\left(\prod_{j=0}^{F-1} s_j^{(\alpha_j - 1)}\right)\left(\prod_{i=1}^{n-k} \gamma_i\right)^{1/2}}{\left(\sum_{j=0}^{F-1} (s_j \beta_j) + \sum_{i=1}^{n-k} (By)_i^2 \gamma_i\right)^{\sum_{j=0}^{F-1} \alpha_j + (n-k)/2}}$$